Module 5: Inverting Functions

TOPIC 3: APPLICATIONS OF EXPONENTIAL FUNCTIONS

In this topic, the term *geometric series* is defined. Students explore different methods to compute any geometric series. They use the pattern generated from repeated polynomial long division to write a formula for the sum of any geometric series. A second formula to compute any geometric series is derived: $S_n = \frac{g_1(r^n - 1)}{r - 1}$. Next, students use their prior knowledge of transformation function form to create graphics on the coordinate plane. Finally, students use iteration and repeated reasoning to explore fractals. Some of the content of this topic goes beyond the scope of the course standards. The content is included to enhance students' understanding of mathematics and provide opportunities for extension.

Where have we been?

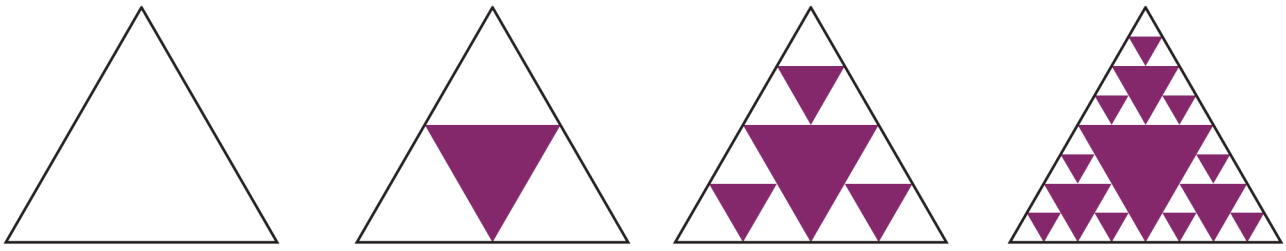
Students have used repeated reasoning and pattern recognition throughout their mathematical careers in both geometry and algebra. They have experience analyzing geometric sequences.

Where are we going?

This topic allows students to flex their creativity in exploring mathematical functions and their transformations. They investigate how applications of iterative operations and repeated reasoning can result in important mathematical products, such as fractals, which have been applied to geographic measurements.

The Sierpinski Triangle

The Sierpinski Triangle is a fractal first described by Polish mathematician Wacław Sierpinski in 1915.



Stage 0 Stage 1 Stage 2 Stage 3

To construct the Sierpinski Triangle:

- Begin with an equilateral triangle.
- Connect the midpoints of the sides, and remove the center triangle by shading it.
- Repeat Stage 1 on the remaining triangles.

The Infinite Cat Project

If you like looking at pictures of cats, then the internet is for you. And if you like looking at pictures of cats looking at pictures of cats on the internet, then look into the Infinite Cat Project.

The project showcased about 1800 pictures of cats looking at pictures of cats looking at . . . you get the (very simple) idea.

Recursion like that featured on the Infinite Cat Project is an interesting mathematical topic and a source of inspiration for self-referential lesson titles.

Talking Points

Geometric series can be an important topic to know about for college admissions tests.

Here is a sample question:

What is the sum of the finite geometric sequence $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$?

One formula to compute a geometric series is given by $S_n = \frac{g_1(r^n - 1)}{r - 1}$, where g_1 is the first term in the series, r is the common ratio, and n is the number of terms. Use the formula to determine the sum:

$$S_n = \frac{2\left(\left(\frac{1}{3}\right)^4 - 1\right)}{\frac{1}{3} - 1} = \frac{80}{27}$$

Key Terms

geometric series

A geometric series is the sum of the terms of a geometric sequence.

fractal

A fractal is a complex geometric shape that is constructed by a repeating mathematical pattern. Fractals are infinite and self-similar across different scales.