

Module 5: Inverting Functions

TOPIC 1: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Students begin this topic by creating exponential graphs using their prior knowledge of geometric sequences. They write exponential growth and decay functions given specified characteristics. The irrational number e , or natural base e , is then introduced through the development of continuous compound interest. Logarithmic functions are introduced as the inverse of exponential functions. Students explore the key characteristics of logarithmic functions and transformations of logarithmic functions, and they state restrictions on the variables for any logarithmic equation.

Where have we been?

In previous courses, students have analyzed exponential functions, including their key characteristics. They have also investigated inverse functions in the previous topic and in earlier courses. Although they may not be familiar with the constant e , students have experience with irrational numbers, including some square roots and π .

Where are we going?

Students will use the intuitions they gain by studying the graphs of logarithmic functions in the next topic to analyze logarithmic equations and apply logarithmic functions to situations. As will be shown in the situations in these topics, logarithmic functions have a number of applications in astronomy, medicine, mechanics, physics, and seismology.

The Triangle of Power

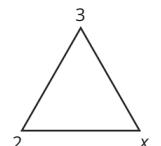
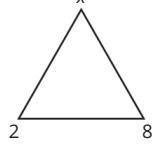
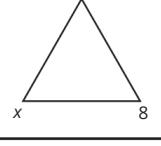
The logarithm of a number for a given base is the exponent to which the base must be raised in order to produce the number. You can use the “Triangle of Power” to help you make sense of logarithms.

logarithm,
or exponent

argument

base

$3 = \log_2 8$

| | |
|--|---------------------------|
|  | $\log_2 x = 3$ $x = 8$ |
|  | $\log_2 8 = x$ $x = 3$ |
|  | $\log_x 8 = 3$ $x = 2$ |

Earthquakes

The Richter scale is used to rate the amount of energy an earthquake releases. This is calculated using information gathered by a seismograph.

The Richter scale is logarithmic, meaning that whole-number jumps in the rating indicate a tenfold increase in the wave amplitude of the earthquake. For example, the wave amplitude in a Level 4 earthquake is ten times greater than the amplitude of a Level 5 earthquake, and the amplitude increases 100 times between a Level 6 earthquake and a Level 8 earthquake.

Only a tiny portion, 15 or so, of the 1.4 million quakes that register above 2 each year register at 7 or above, which is the threshold for an earthquake to be considered major.

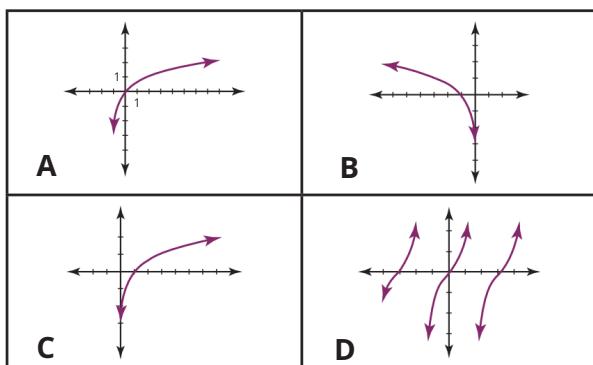


Talking Points

Logarithmic functions can be an important topic to know about for college admissions tests.

Here is a sample question:

Identify the graph of $f(x) = \ln(-x)$.



The function $f(x) = \ln(-x)$ is a reflection of the function $f(x) = \ln(x)$ across the y -axis.

Choice B is correct.

Key Terms

natural base e

The natural base e is a mathematical constant approximately equal to 2.71828.

logarithm

The logarithm of a number for a given base is the exponent to which the base must be raised in order to produce the number.

natural logarithm

A natural logarithm is a logarithm with base e , and it is usually written as "ln."