

# Module 4: Extending Beyond Polynomials

## TOPIC 2: RADICAL FUNCTIONS

This topic presents opportunities for students to explore radical functions, rewrite radical expressions, and solve radical equations. Using patty paper, students switch the  $x$ - and  $y$ -axes of power functions and generate their inverses, recognizing that they are transposing the axes when they are inverting a function. Students then shift their focus from the graphical representation of inverses to the algebraic representation. With an understanding of radical functions, students then consider radical expressions and equations.

## Where have we been?

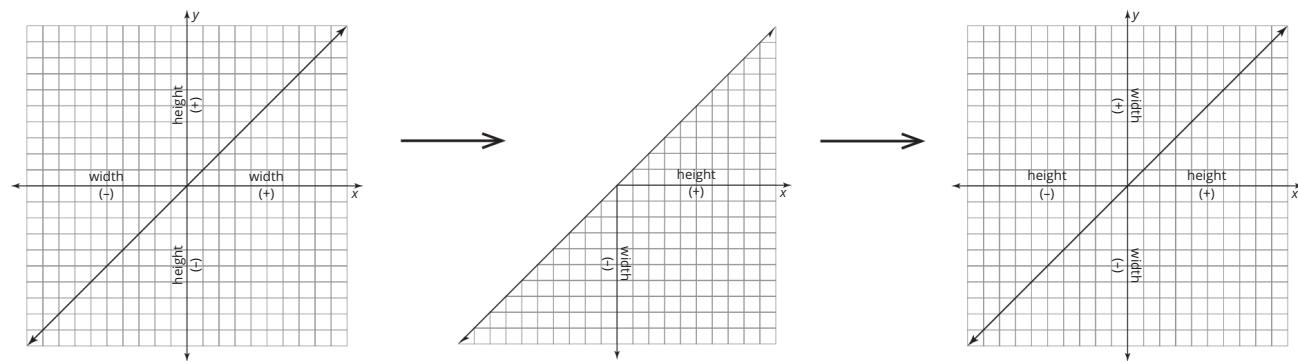
Beginning in middle school, students have been solving equations using the Properties of Equality. They solved for unknown values in the Pythagorean Theorem by taking the square root of both sides. They have experience with reflecting functions across the line  $y = x$  to determine an inverse graphically and transposing  $x$  and  $y$  in an equation to determine an inverse algebraically. They have used exponential functions to reason why  $x^{\frac{m}{n}} = \sqrt[n]{x^m}$  and have extracted perfect squares from under radicands.

## Where are we going?

Students will use their understanding of function inverses throughout the next module. They will explore the inverses of exponential functions, which introduces them to logarithmic functions. Radical functions are used extensively to model real-world problems, particularly in physics and in other applications, including medical dosage, wind speed, pendulums, and centrifugal force.

## Graphs of Inverses

The graph of the inverse of a function is a reflection of the graph across the line  $y = x$ .



The inverse of the linear function  $f(x) = x$  is the same line.

## Transpositions

The word *transpose* means *to switch two or more items*. The word combines the Latin prefix *trans-*, meaning “across” or “over” and *ponere*, meaning “to put” or “place.” The word *interchange* means the same thing as transpose.

Like many words, *transpose* is used in different ways in different fields:

- In music, the word transpose is most often used to mean rewriting a song in a different key—either higher or lower.
- In biology, a transposable element is a sequence of DNA that can move from one location to another in a gene.
- Magicians use transposition when they make two objects appear to switch places.

Keep an eye out for the word *transpose* in these lessons.

## Talking Points

Composition of functions can be an important topic to know about for college admissions tests.

Here is a sample question:

**Let  $g(x) = x^2 - 5$ . If  $f(g(x)) = \sqrt{x^2 + 4}$ , what is  $f(x)$ ?**

The entire expression  $x^2 - 5$  is the input of the function  $f$ , and the output is  $\sqrt{x^2 + 4}$ , so what does  $f$  do to the input to produce the output?

If you add 9 to  $x^2 - 5$ , and then take the square root of that, you get the correct output, so the function  $f$  adds 9 and takes the square root of its input.

Thus,  $f(x) = \sqrt{x + 9}$ .

## Key Terms

### inverse of a function

The inverse of a function is the set of all ordered pairs  $(y, x)$ , or  $(f(x), x)$ .

### radical function

The inverses of power functions with exponents greater than or equal to 2, such as the square root function and the cube root function, are called radical functions.

### composition of functions

The process of evaluating one function inside of another function is called the composition of functions. For two functions  $f$  and  $g$ , the composition of functions uses the output of  $g(x)$  as the input of  $f(x)$ .