# Module 2: Developing Structural Similarities

### TOPIC 1: RELATING FACTORS AND ZEROS

In this topic, the Factor Theorem is introduced and used to determine if a linear expression is a factor of a polynomial function. Students then perform polynomial long division, and the Remainder Theorem is stated and used to answer questions involving polynomial division with remainders. Methods of factoring polynomials are introduced. Finally, students investigate the Closure Property for polynomials.

## Where have we been?

Students have factored mathematical expressions since elementary school. In previous courses, they have factored degree-2 polynomial equations in order to isolate key characteristics of the functions represented by those equations. Students are also familiar with the long division algorithm from elementary school, used in this topic in a similar way to divide polynomials.

## Where are we going?

Polynomial equations are used extensively in industry to track financial and inventory information, in data science to build predictive models, and to help analyze and answer research questions. Students will use what they learned in this topic to help them build real-world polynomial models in the next topic.

## Long Division with Polynomials

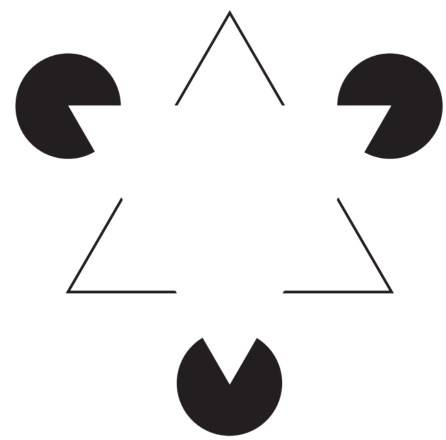
You can use long division with polynomials. Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. The process is similar to integer long division.

| Integer Long Division | Polynomial Long Division |
|---|---|
| $3660 \div 12$<br>or<br>$\frac{3660}{12}$<br>$\begin{array}{r} 305 \\ 12\overline{)3660} \\ -36 \\ \hline 6 \\ -0 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$ | $(x^3 + 12x^2 + 41x + 72) \div (x + 8)$ or $\frac{x^3 + 12x^2 + 41x + 72}{x + 8}$<br>$\begin{array}{r} \text{(A)}\,x^2 + \text{(D)}\,4x + \text{(G)}\,9 \\ x + 8\overline{)x^3 + 12x^2 + 41x + 72} \\ \text{(B)}\,-(x^3 + 8x^2)\downarrow \qquad \\ \hline \text{(E)}\,4x^2 + 41x\ \text{(C)} \quad \\ -(4x^2 + 32x)\downarrow \\ \hline 9x + 72\ \text{(F)} \\ \text{(H)}\,-(9x + 72) \\ \hline \end{array}$<br>Remainder 0<br>A. Divide $\frac{x^3}{x} = x^2$.<br>B. Multiply $x^2(x + 8)$ and then subtract.<br>C. Bring down $41x$.<br>D. Divide $\frac{4x^2}{x} = 4x$.<br>E. Multiply $4x(x + 8)$ and then subtract.<br>F. Bring down $+72$.<br>G. Divide $\frac{9x}{x} = 9$.<br>H. Multiply $9(x + 8)$ and then subtract. |

## Getting Closure

The word *closure* can mean many things depending on the context. For instance, *closure* for a business may be caused by an organization going bankrupt. In government, *closure*, which is also referred to as *cloture*, is a procedure by which the Senate can vote to place a time limit on consideration of a bill.

*Closure* may also refer to humans' ability to perceive objects as wholes, even when some of the parts are missing. Your brain fills in the missing parts. For example, you perceive a white triangle on the right, even though it is not drawn there at all.



## Talking Points

Polynomial division can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

**For what value of the constant $p$ does $\frac{6x^4 + 2x^2 - 8x - p}{x + 2}$ have no remainder?**

When you use long division to divide the polynomials, you see that the quotient is $6x^3 - 12x^2 + 26x - 60$ with a remainder of $-p + 120$.

Thus, the quotient will have no remainder only if $p$ has a value of 120.

## Key Terms

**Factor Theorem**

The Factor Theorem states that a polynomial function $p(x)$ has $(x - r)$ as a factor if and only if the value of the function at $r$ is 0, or $p(r) = 0$.

**Remainder Theorem**

The Remainder Theorem states that when any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$, or the value of the equation or function when $x = r$.

**closed under an operation**

When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be closed under that operation.