Module 2: Analyzing Structure

TOPIC 1: COMPOSING AND DECOMPOSING FUNCTIONS

This topic introduces students to the concept of building new functions on the coordinate plane by operating on or translating functions. Students review the modeling process—Notice and Wonder, Organize and Mathematize, Predict and Analyze, and Test and Interpret—which can be used to solve real-world problems. Familiar with translating and dilating functions by a constant, students explore the higher-degree functions that result when translating and dilating functions by non-constant amounts. After building functions from linear and quadratic factors, students decompose functions.

Where have we been?

Beginning in middle school, students have studied the key characteristics of linear functions, including the *x*- and *y*-intercept and slope. In earlier high school courses, students examined the graphs of linear and quadratic functions and learned that the *x*-intercepts of a graph are the zeros of the function. They have used the linear factors of a quadratic function to demonstrate the Zero Product Property and have used this property to solve quadratic equations.

Where are we going?

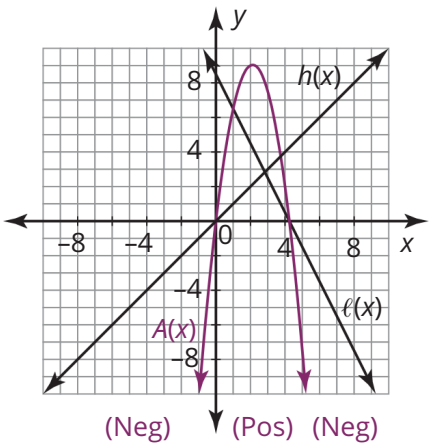
In the upcoming topics in this course, students will continue to analyze the structure of polynomial functions. Understanding zeros graphically lays the foundation for understanding and solving for zeros algebraically, and the connection to graphical representations of functions helps students to factor polynomials using long division or synthetic division.

Building Functions

You can use the product of linear functions to build a polynomial. The graph shows the product of two functions, $h(x) \cdot l(x) = A(x)$.

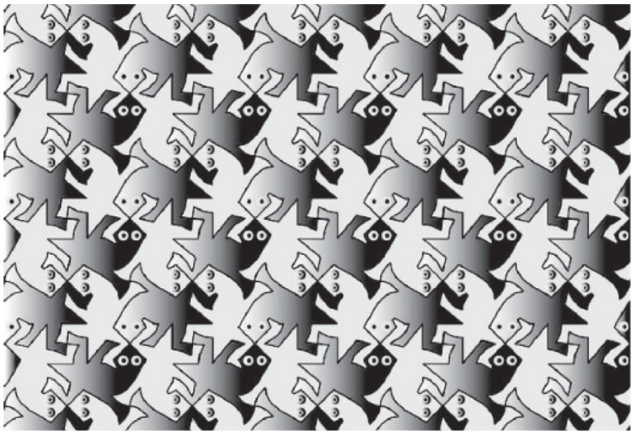
The new function crosses the *x*-axis at each of its factor's *x*-intercepts.

The graph increases or decreases depending on whether the output is positive or negative as it moves from one interval to the next.



Escher Time

M.C. Escher was a well-known artist with a unique visual perspective. Many of his works display elusive connections, peculiar symmetry, and tessellations. Tessellations are symmetric designs with a repeated pattern. You can find many images of Escher's work on the World Wide Web. Take a look and enjoy! Make sure to take a close look, because things may not be as straightforward as they seem.



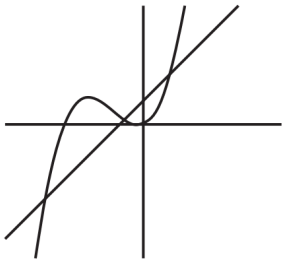
Talking Points

Cubic functions can be an important topic to know about for college admissions tests.

Here is an example of a sample question:

The graph of a cubic function and the graph of $y = x + 1$ in the xy-plane can intersect in at most how many points?

You can visualize the linear function and a cubic function as shown:



At most, there are 3 intersection points.

Key Terms

cubic function

A cubic function is a function that can be written in the standard form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. A cubic function is a polynomial function of degree 3.

multiplicity

Multiplicity is how many times a particular number is a zero for a given function.

Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra states that a degree-n polynomial has, counted with multiplicity, exactly n zeros.