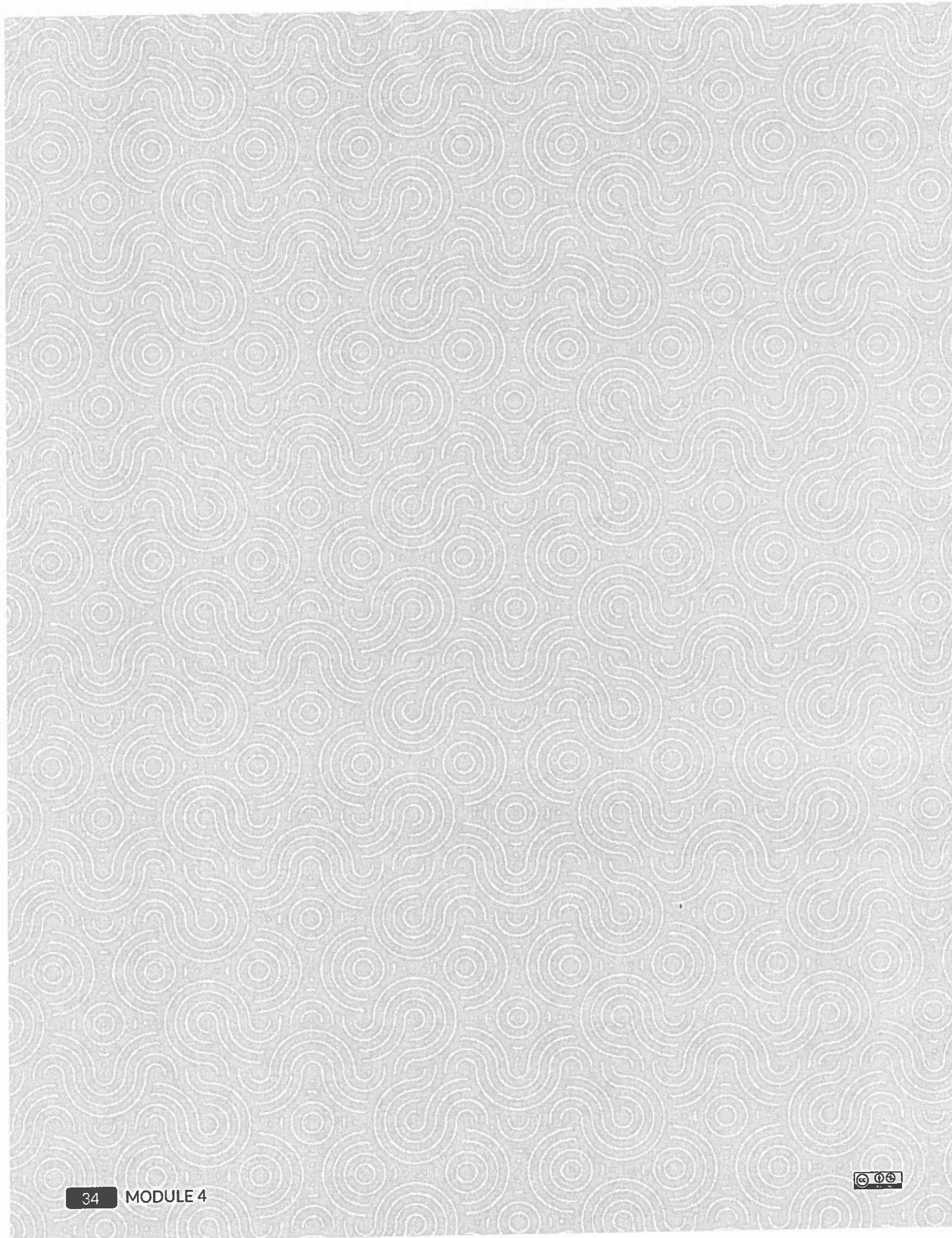


Investigating Growth and Decay

TOPIC 1	Introduction to Exponential Functions	35
TOPIC 2	Using Exponential Equations	39





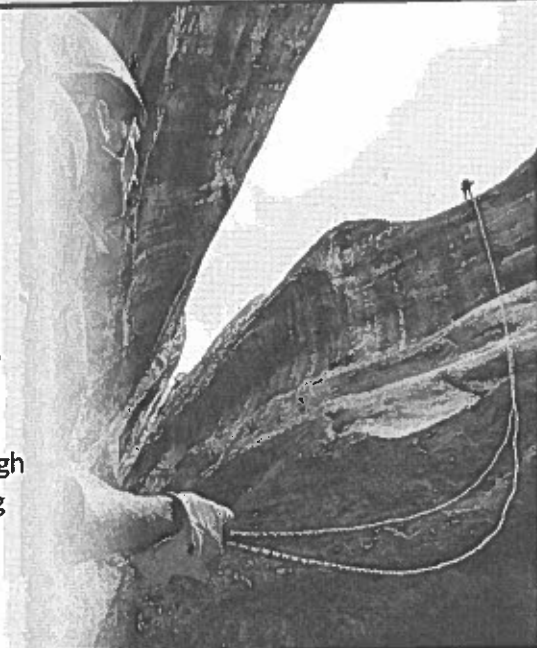
Family Guide

MODULE 4 Investigating Growth and Decay

Algebra 1

TOPIC 1 Introduction to Exponential Functions

At the start of this topic, students learn and apply properties of integer exponents to simplify expressions. Next, students build upon their previous understanding of sequences and common ratios to recognize that some geometric sequences are exponential functions and others are not. Then, students examine the structure of exponential functions, connecting the common ratio of a geometric sequence with the base of the power in an exponential function. Students explore the constant ratio between intervals of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ for the function $f(x) = 2^x$. Through this exploration, they conclude that $2^{\frac{1}{2}} = \sqrt{2}$ and $2^{\frac{1}{3}} = \sqrt[3]{2}$. By graphing these values, the misconception that $f(\frac{1}{2})$ is halfway between $f(0)$ and $f(1)$ is addressed. Finally, students learn and apply properties of rational exponents to simplify expressions.



Where have we been?

Students have learned to write and evaluate numerical and algebraic expressions with whole number exponents in middle school. This topic expands on that knowledge. Earlier in this course, students were introduced to geometric sequences. Students can already write a recursive and an explicit formula for a given geometric sequence.

Where are we going?

- Throughout this topic, students apply what they know about the key characteristics of functions (e.g., intercepts, intervals of increase or decrease, and domain and range) to include exponential functions. This prepares them for the work they will do with quadratic functions, both in this course and with more complex functions in future courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Exponential functions are an important topic to know about for post-secondary readiness, including college admission tests.

HERE IS A SAMPLE QUESTION

A biology class predicted that a population of animals will double in size every year. The population at the start of 2024 was about 500 animals. If P represents the population n years after 2024, what equation represents the model of the population over time?

To solve this, students should know that this represents an exponential function because the population doubles each year. This can be written as:

$$\text{initial value} \cdot (\text{growth rate})^{\text{time}}$$

The initial value is 500 animals, and the growth rate is 2, for doubling. So, the function $P(n) = 500 \cdot 2^n$ models the population over time in years, n .

NEW KEY TERMS

- power [potencia]
- base [base]
- exponent [exponente]
- horizontal asymptote [asíntota horizontal]
- perfect square
- square root
- radical [radical]
- radicand [radicando]
- product property of radicals [propiedad del producto de radicales]
- extracting perfect squares
- index [índice]
- quotient property of radicals [propiedad del cociente de radicales]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **power** has two elements: the base and the exponent.

base $\rightarrow 6^2 \leftarrow$ exponent
power

The **base** of a power is the factor that is multiplied repeatedly in the power.

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

↑
base

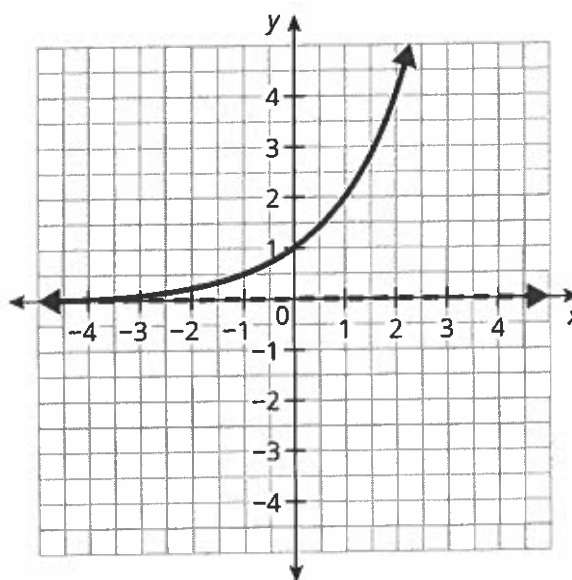
The **exponent** of the power is the number of times the base is used as a factor.

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

↑
exponent

A **horizontal asymptote** is a horizontal line that a function gets closer and closer to, but never intersects.

The graph shows a horizontal asymptote at $y = 0$.



In **Lesson 1: Properties of Powers with Integer Exponents**, students write and evaluate expressions with positive integer exponents.

Power Properties

Powers have properties that can be used to expand, simplify, or just rewrite them differently. These properties are important in later math classes and students must be able to remember them because they are useful tools in algebra.

Properties of Powers	Words	Rule
Product of powers	To multiply powers with the same base, keep the base and add the exponents.	$a^m a^n = a^{m+n}$
Power of a power rule	To simplify a power of a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient of powers rule	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$
Zero power	The zero power of any number, except for 0, is 1.	$a^0 = 1$, if $a \neq 0$
Negative exponents in the numerator	An expression with a negative exponent in the numerator and a 1 in the denominator equals 1 divided by the power with its opposite exponent placed in the denominator.	$a^{-m} = \frac{1}{a^m}$, if $a \neq 0$ and $m > 0$
Negative exponents in the denominator	An expression with a negative exponent in the denominator and a 1 in the numerator equals the power with its opposite exponent.	$\frac{1}{a^{-m}} = a^m$, if $a \neq 0$ and $m > 0$

In **Lesson 3: Geometric Sequences and Exponential Functions**, students revisit geometric sequences as a launch to exponential functions.

Constant Ratios

Students make the connection between a geometric sequence and an exponential function of the form $f(x) = ab^x$. They prove with algebra that there is always a constant ratio from one output value to the next of an exponential function.

The sequence 3, 6, 12, 24 is a geometric sequence with a common ratio of 2 because in each step, the value is multiplied by 2.



MYTH

Cramming for a test is just as good as spaced practice for long-term retention.

Everyone has been there. You have a big test tomorrow, but you've been so busy that you haven't had time to study. So, you had to learn it all in one night. You may have received a decent grade on the test. However, did you remember the material a week, a month, or a year later?

The honest answer is, "probably not." That's because long-term memory is designed to retain useful information. How does your brain know if a memory is "useful" or not? One way is the frequency in which you encounter a piece of information. If you see something only once (like during cramming), then your brain doesn't deem those memories as important. However, if you sporadically come across the same information over time, your brain is more likely to recognize it as important. To optimize retention, encourage your student to periodically study the same information over expanding intervals of time.

#mathmythbusted

In **Lesson 4: Rewriting Square Roots**, students use the Properties of Radicals to simplify square root expressions.

In **Lesson 5: Rational Exponents and Graphs of Exponential Functions**, students learn that the Properties of Powers apply to expressions with rational exponents, rewrite expressions with rational exponents as radicals, and connect these two concepts to perform and justify operations involving radicals.

Extracting Perfect Squares

Students will be able to rewrite radical expressions by **extracting perfect squares**. This is the process of removing perfect squares from under the radical symbol. In the example below, the number 450 is broken down into its factors, which are the numbers that can be multiplied to equal 450. This is done using division. When 450 has been divided to form its prime factorization, the numbers 3 and 5 appear two times, which means that they can be combined into perfect squares and removed from under the radical symbol. This is because $\sqrt{3 \cdot 3} = \sqrt{3^2} = 3$. After removing the 3 and 5, they are multiplied to equal 15, and the factor 2 stays under the radical symbol.

$$\begin{aligned}\sqrt{450} &= \sqrt{2 \cdot 225} \\ &= \sqrt{2 \cdot 3 \cdot 75} \\ &= \sqrt{2 \cdot 3 \cdot 3 \cdot 25} \\ &= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} \\ &= \sqrt{2 \cdot 3^2 \cdot 5^2} \\ &= 3 \cdot 5\sqrt{2} \\ &= 15\sqrt{2}\end{aligned}$$



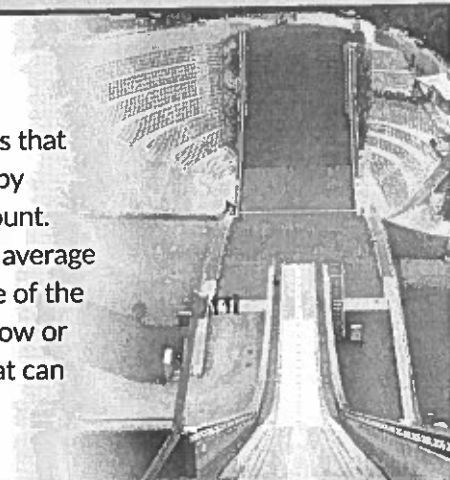
Family Guide

MODULE 4 Investigating Growth and Decay

Algebra I

TOPIC 2 Using Exponential Equations

In this topic, students explore strategies for distinguishing exponential functions that represent growth scenarios versus those that represent decay. Students begin by comparing the value of a simple interest account and a compound interest account. They graph and write equations for these two scenarios and then compare the average rate of change of each for a given interval. Students then examine the structure of the exponential equations to recognize scenarios in which exponential functions grow or decay. Throughout the rest of the topic, students solve real-world problems that can be modeled by exponential functions. Students use technology to generate exponential regressions and use those functions to make predictions.



Where have we been?

Students know the rules of exponents and are familiar with the structure of exponential functions from their work in the previous topic. This topic is an opportunity to reinforce their understanding and build upon their skills by recognizing and solving problems that can be modeled by exponential functions. In previous courses, students analyzed bivariate data looking for linear, nonlinear, or no association and used trend lines to make predictions. Previously in this course, they used technology to generate linear regressions and analyzed the correlation coefficient for fit to the data set. In this topic, students use technology to generate exponential regressions to introduce a new function to fit the data.

Where are we going?

This topic represents students' first deep dive into solving equations that represent nonlinear functions. As students gain proficiency in solving increasingly complex equations, they are able to model more interesting and complex real-world phenomena.

TALKING POINTS

Discuss with your student Exponential functions represent real-world situations such as compound interest and bacterial growth. They are also important to know for college admissions tests.

HERE IS A SAMPLE QUESTION

A car valued at \$21,000 depreciates at a rate of 17% per year. What is the value of the car after 5 years?

To solve, students should know to use the model for exponential decay, $y = a(1 - r)^x$, where a represents the initial value, r represents the rate of decrease, and x represents time.

$$\begin{array}{lcl} y = a(1 - r)^x & : & y = 21,000(0.83)^5 \\ y = a(1 - 0.17)^x & : & y = 8271.99 \end{array}$$

In 5 years, the car will be worth \$8271.99.

NEW KEY TERMS

- simple interest [interés simple]
- compound interest [interés compuesto]
- exponential growth function
- exponential decay function [función de decaimiento exponencial]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

An **exponential growth function** is an exponential function with a b -value greater than 1 and is of the form $y = a(1 + r)^x$, where r is the rate of growth.

An **exponential decay function** is an exponential function with a b -value greater than 0 and less than 1 and is of the form $y = a(1 - r)^x$, where r is the rate of decay.

In **Lesson 1: Exponential Equations for Growth and Decay**, students compare linear and exponential functions in the context of simple interest and compound interest situations. They then identify the values in the exponential function equation that indicate whether it is a growth or decay function and apply this reasoning in context.

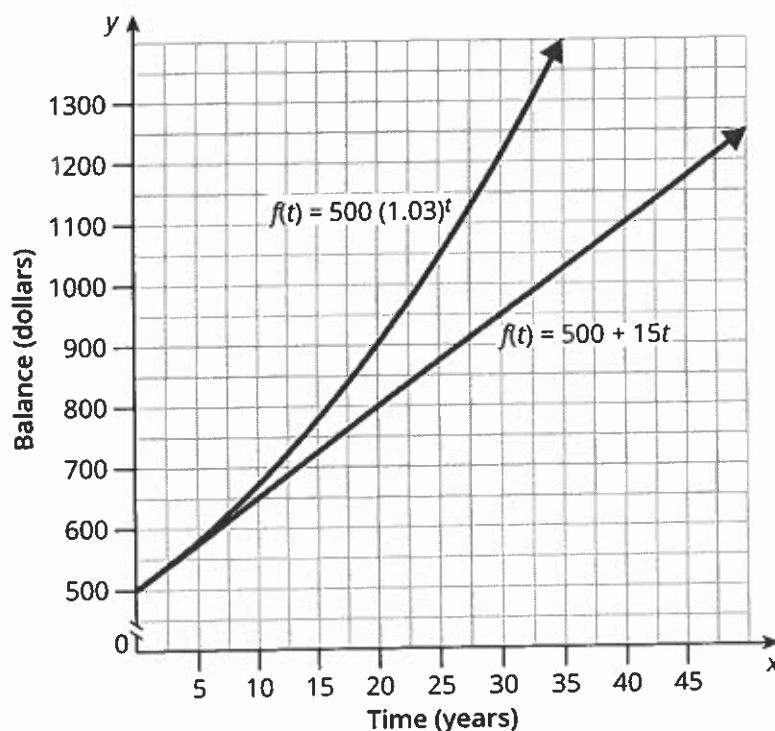
Simple and Compound Interest

Learning about simple and compound interest teaches students about saving and borrowing money. With **simple interest**, money grows at the same rate over time, while with **compound interest** money grows at a faster rate as time passes. Knowing how percentage growth makes money grow faster and faster is the foundation for building wealth.

Time (years)	Simple Interest Balance (dollars)	Compound Interest Balance (dollars)
0	500	500
1	515	515
2	530	530.45
10	650	671.96
100	2000	9609.32

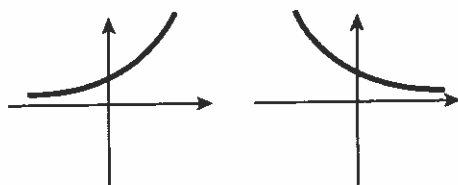
$$f(t) = 500(1.03)^t$$

- $a = 500$
- $b = 1.03$
- $r = 0.03$



Exponential Growth and Decay

Some things in nature, like bacteria, grow faster as time passes. This is called *exponential growth*. In the same way, some things like a healing wound will become smaller or shrink faster as time passes. This is called *exponential decay*. You can use the exponential function $f(x) = ab^x$ to model exponential growth and exponential decay. An exponential growth function is in the form $f(x) = a(1 + r)^x$, where r is the rate of growth. An exponential decay function is in the form $f(x) = a(1 - r)^x$, where r is the rate of decay.

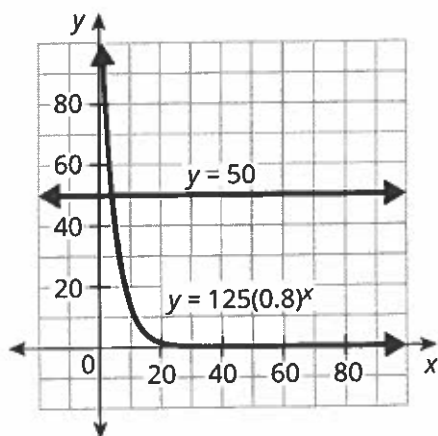


In **Lesson 2: Interpreting Parameters in Context**, students recall how to solve an equation graphically by graphing both sides of the equation and determining the point of intersection.

Solving Exponential Equations by Graphing

Graphs can be used to solve exponential equations by estimating the intersection point of the graph of an exponential function with a constant function.

For example, to determine the solution to $125(0.8)^x = 50$, graph each side of the equation as separate functions on the same coordinate plane.



The solution to the equation is $x \approx 4$.

In **Lesson 3: Modeling Using Exponential Functions**, students create a scatterplot, write a regression equation, use the function to calculate output values, and interpret the reasonableness of a prediction based on the scenario.



MYTH

"I'm not smart."

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart"? On one hand, a baby is helpless and doesn't know anything. But on the other hand, a baby is insanely smart because they are constantly learning new things every day!

This example is meant to demonstrate that *smart* can have two meanings. It can mean *the knowledge that you have*, or it can mean *the capacity to learn from experience*. When someone says they are "not smart," are they saying they do not have much knowledge, or are they saying that they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know this is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort, but the brain is automatically wired to learn. Smart should not refer only to how much knowledge you currently have.

#mathmythbusted

Exponential Regression

Students learn that just as some data points almost match the shape of a straight line, others match more closely to a curve that is made by an exponential function. This makes it possible to make predictions about things that grow exponentially in the same way that we can with linear data.

Year Since 1985	Number of Cell Phone Users
0	285
1	498
3	1527
4	2672
6	8186
7	14,325
8	25,069

