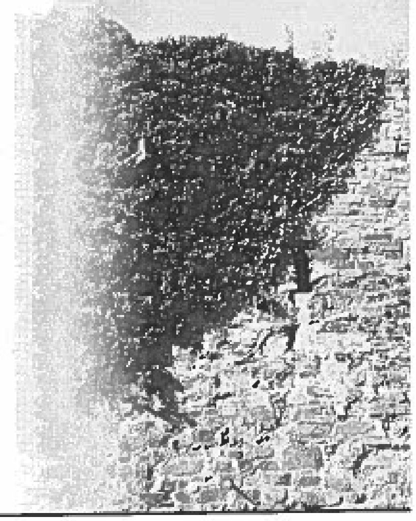
MODULE 3

Modeling Linear Equations and Inequalities

TOPIC 1 Linear Equations and Inequalities



In this topic, students analyze linear functions and the key characteristics that define linear functions. They solve equations in one variable, examining the structure of each equation to predict whether the equation has one solution, no solution, or infinite solutions. Students use the properties of equality and basic number properties to construct a viable argument to justify a solution method. They generalize their knowledge of solving equations in one variable to solve literal equations for given variables. Students then graph linear inequalities and explore solving an inequality with a negative slope, which affects the sign of the inequality.

Where have we been?

In previous courses, students gained proficiency in solving increasingly complex linear equations and they solved two-step inequalities and graphed the solutions on a number line. Entering this course, students have solved two-step equations with variables on both sides. They understand the underpinnings of solving equations by maintaining equality. From this intuitive understanding, students use properties to justify each step in the equation-solving process.

Where are we going?

Students will use their knowledge of equations and inequalities to solve linear absolute value equations in future courses. By recognizing the connections between algebraic and graphical solutions to an equation or inequality, students are developing the foundation to later solve linear absolute value equations and inequalities, exponential equations, and quadratic equations and inequalities.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Inequalities can be an important topic to know about for college admissions tests.

HERE IS A SAMPLE QUESTION

Solve for x in the inequality $\frac{x}{2} - 3 < 2y$.

To solve for x, isolate the variable x.

$$\frac{x}{2} - 3 < 2y$$
$$\frac{x}{2} - 3 + 3 < 2y + 3$$
$$\frac{x}{2} < 2y + 3$$
$$\cancel{2}\left(\frac{x}{\cancel{2}}\right) < 2(2y + 3)$$
$$x < 4y + 6$$

NEW KEY TERMS

- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

$5x - 7 = 13$

$5x - 7 + 7 = 13 + 7$

$5x = 20$

$x = 4$

One solution

$5x - 7 = 3 + 5x - 10$

$5x - 7 = 5x - 7$

$5x - 7 - 5x = 5x - 7 - 5x$

$-7 = -7$

Infinite solutions

$5x - 7 = 5x - 10$

$5x - 7 - 5x = 5x - 10 - 5x$

$-7 = -10$

$-7 \neq -10$

No solution

Generally, there is only one **solution** to an equation. However, students will discover that there are special cases where an equation will have either **no solution** or **infinite solutions** and identify what those cases look like.

In **Lesson 1: Solving Linear Equations**, students review and use the properties of equality and basic number properties to solve linear equations and justify their steps. Justifying steps with the properties of equality will better prepare students for writing geometrical proofs in future courses.

Properties of Equality

The properties of equality state that when an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality.

Properties of Equality	For all numbers *a*, *b*, and *c*
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

There are also basic number properties that can be used to justify steps when solving equations.

Number Properties	For all numbers *a*, *b*, and *c*
commutative property	$a + b = b + a$ $ab = ba$
associative property	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$
distributive property	$a(b + c) = ab + ac$

In **Lesson 2: Literal Equations**, students solve literal equations for specified variables. This connects back to students' prior knowledge of area, volume, and surface area as they solve for unknown values in formulas.

Literal Equations

Literal equations are like solving equations, but instead of having a singular number answer, it is generally a manipulation of an equation to solve for a single variable. For example, students already know how to determine the area of a triangle. They can use the formula for area of a triangle, $A = \frac{1}{2}bh$, to solve for the base or the height of a triangle. Manipulating the known equation can make calculations easier.

$$A = \frac{1}{2}bh$$
$$2(A) = \cancel{2}\left(\frac{1}{\cancel{2}}bh\right)$$
$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$
$$\frac{2A}{b} = h$$

In **Lesson 3: Modeling Linear Inequalities**, students model **linear inequalities** with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement.

Solutions to Inequalities

Sometimes, it is necessary to express that more than one answer could satisfy a situation. Students will learn that phrases like *at least*, *at most*, *no more than*, and *less than* will create situations where many answers may be correct.

$>$	$<$
greater than	less than
$\geq$	$\leq$
greater than or equal to	less than or equal to

Solving Inequalities

To solve an inequality, first write a function to represent the problem situation. Then, write the function as an inequality based on the independent quantity. To determine the solution, identify the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: isolate the variable on one side of the inequality symbol. Finally, interpret the meaning of the solution.

For example, consider the situation in which Diego has $25 in his gift fund that he is going to use to buy graduation gifts. Graduation is 9 weeks away. If he would like to have at least $70 to buy gifts, how much should he save each week?

The function is $f(x) = 25 + 9x$, so the inequality is $25 + 9x \geq 70$.

$$\begin{aligned} 25 + 9x &\geq 70 \\ 25 + 9x - 25 &\geq 70 - 25 \\ 9x &\geq 45 \\ \frac{9x}{9} &\geq \frac{45}{9} \\ x &\geq 5 \end{aligned}$$

Diego needs to save at least $5 each week to meet his goal.

When you multiply or divide each side of an inequality by a negative number, the inequality sign reverses. Consider the division example in the inequality $250 - 9.25x < 398$.

$$\begin{aligned} 250 - 9.25x &< 398 \\ 250 - 9.25x - 250 &< 398 - 250 \\ -9.25x &< 148 \\ \frac{-9.25x}{-9.25} &> \frac{148}{-9.25} \\ x &> -16 \end{aligned}$$

MYTH

Just watch a video, and you will understand it.

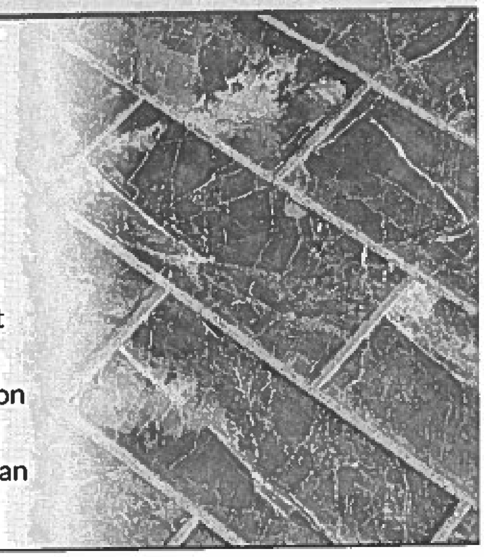
Has this ever happened to you? Someone explains something, and it all makes sense at the time. You feel like you get it. But then, a day later when you try to do it on your own, you suddenly feel like something's missing? If that feeling is familiar, don't worry. It happens to us all. It's called the illusion of explanatory depth, and it frequently happens after watching a video.

How do you break this illusion? The first step is to try to make the video interactive. Don't treat it like a TV show. Instead, pause the video and try to explain it to yourself or to a friend. Alternatively, attempt the steps in the video on your own and rewatch it if you hit a wall. Remember, it's easy to confuse familiarity with understanding.

#mathmythbusted

Family Guide

MODULE 3 Modeling Linear Equations and Inequalities Algebra I



TOPIC 2 Systems of Equations and Inequalities

In this topic, students begin with writing systems of linear equations and solving them graphically and algebraically using substitution. They then move on to solve systems of linear equations using the linear combinations method. Students consider linear inequalities in two variables and learn that their solutions are represented as half-planes on a coordinate plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes. Finally, students synthesize their understanding of systems by encountering problems that can be solved by using either a system of equations or a system of inequalities.

Where have we been?

Coming into this topic, students know that every point on the graph of an equation represents a value that makes the equation true. They have learned that the point of intersection of two graphs provides x- and y-values that make both equations true. Students have written systems of linear equations and have solved them graphically.

Where are we going?

Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In future courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Systems of equations is an important topic to know for modeling real-world situations such as supply and demand or profit and cost.

HERE IS A SAMPLE QUESTION

If (x, y) is a solution to the system of equations, what is the value of $x - y$?

$$\begin{aligned} 2x - 3y &= -14 \\ 3x - 2y &= -6 \end{aligned}$$

Multiplying the first equation by 3 and the second equation by -2 gives

$$\begin{aligned} 6x - 9y &= -42 \\ -6x + 4y &= 12 \end{aligned}$$

Then, adding the equations gives

$$\begin{aligned} -5y &= -30 \\ y &= 6 \end{aligned}$$

The value of y can be substituted in one of the equations to get the value of x.
The solution is $(4, 6)$, so $x - y = -2$.

NEW KEY TERMS

- system of linear equations [sistema de ecuaciones lineales]
- consistent systems [sistemas consistentes]
- inconsistent systems [sistemas inconsistentes]
- standard form of a linear equation [forma estándar/genera de una ecuación lineal]
- substitution method [método de sustitución]
- linear combinations method [método de combinaciones lineales]
- half-plane
- boundary line
- constraints
- solution of a system of linear inequalities [solución de un sistema de desigualdades lineales]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

The **standard form of a linear equation** is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

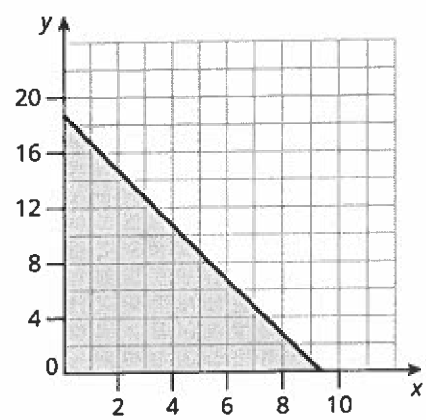
When two or more linear equations define a relationship between quantities, they form a **system of linear equations.**

The equations $y = 3x + 7$ and $y = -4x$ are a **system of linear equations.**

$$\begin{cases} y = 3x + 7 \\ y = -4x \end{cases}$$

The graph of a linear inequality is a **half-plane**, or half of a coordinate plane.

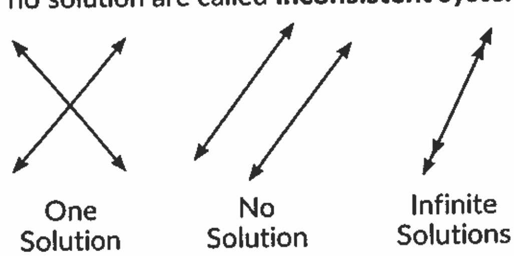
The shaded portion of the graph is a half-plane.



In **Lesson 1: Using Graphing to Solve Systems of Equations**, students write systems of linear equations and solve them graphically.

Solving Systems of Linear Equations by Graphing

The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system is to graph both equations and identify the point at which the two graphs intersect. A system of equations may have no solution, one unique solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.



In **Lesson 2: Using Substitution to Solve Linear Systems**, students explore one of the two algebraic methods to solve a system.

Solving Systems of Linear Equations by Substitution

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Let's consider this system:

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. This will be considered the first equation.

Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$, and write the new equation.

$$1.25x + 1.05(8x) = 30$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$1.25x + 8.40x = 30$$

$$9.65x = 30$$

$$x \approx 3.1$$

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

The solution to the system is about (3.1, 24.8).

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

MYTH

"Just give me the rule. If I know the rule, then I understand the math."

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can, because now it makes sense.

Learning does not take place in a vacuum. It ***must be*** connected to what you already know. Otherwise, arbitrary rules will be forgotten.

#mathmythbusted

In **Lesson 3: Using Linear Combinations to Solve a System of Linear Equations**, students explore another method to solve a system algebraically.

Solving Systems of Linear Equations by Linear Combinations

The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{array}{r} 7x + 2y = 24 \\ + \quad -8x - 2y = -30 \\ \hline -x = -6 \\ x = 6 \end{array}$$

Now that the y-values are additive inverses, you can add the equations and solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

In **Lesson 5: Systems of Linear Inequalities**, students graph a system of linear inequalities to determine possible solutions to the system.

Systems of Linear Inequalities

Often problem situations offer more than one solution to them. Students are going to use graphs of these problem situations that are expressed as systems of linear inequalities to identify solutions and non-solutions by recognizing key areas of the graphs.