MODULE 2

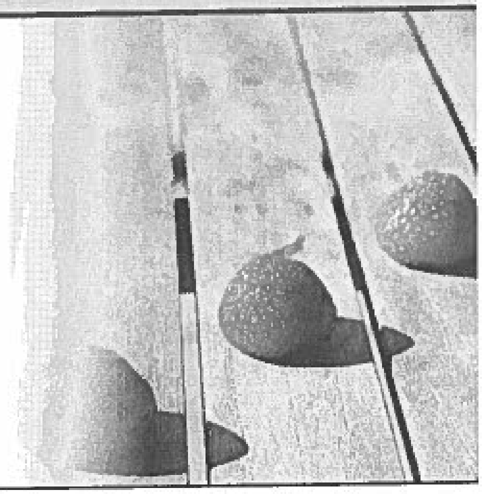
Exploring Constant Change

Family Guide

MODULE 2 Exploring Constant Change

Algebra I

TOPIC 1 Linear Functions



In this topic, students focus on the patterns that are evident in certain data sets and use linear functions to model those patterns. Using the informal knowledge of lines of best fit that was built in previous grades, students advance their statistical methods to make predictions about real-world phenomena.

Students prove that the common difference of an arithmetic sequence and the slope of the corresponding linear function are both constant and equal. They write equations and graph lines presented in slope-intercept, point-slope, and standard forms.

Where have we been?

Students have analyzed the shape of data, informally fit trend lines to model data sets, determined the equations of those lines, interpreted the slopes and y-intercepts of the lines, and used the equations to make and judge the reasonableness of predictions about the data. Students have also examined linear relationships and recognized that the slope of a line defines its steepness and direction.

Where are we going?

As students continue in this course and in future mathematics courses, they will determine and analyze more complicated regressions, including exponential and quadratic regression models. From this topic, students should understand the key and defining characteristics of a linear function represented in situations, tables, equations, and graphs. This prepares students exploring equations as the most specific representation of linear functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Regressions can be an important topic to know about for modeling real-world data and for college admissions tests.

HERE IS A SAMPLE QUESTION

The data in the table show test scores after certain amounts of study time. Use a linear function to estimate the score associated with a study time of 20 minutes.

Score	86	70	90	78
Time (min)	45	15	40	35

Time is the independent variable and score is the dependent variable. Use graphing technology to determine a linear regression function.

This yields a linear regression function of $f(x) = 0.61x + 60.51$. A study time of 20 minutes would yield an estimated score of $f(20) = 0.61(20) + 60.51$, or 72.71.

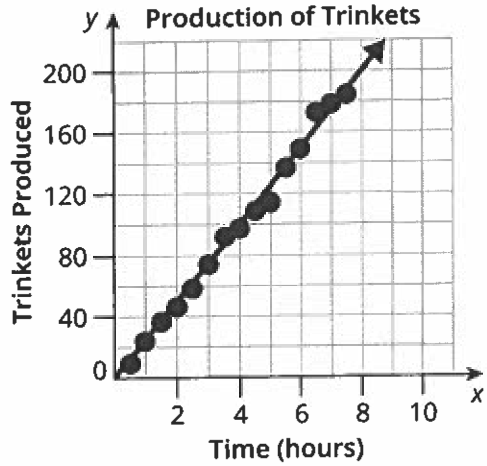
NEW KEY TERMS

- Least Squares Method
- centroid [centroide]
- linear regression function [función de regresión lineal]
- interpolation [interpolación]
- extrapolation [extrapolación]
- correlation [correlación]
- correlation coefficient [coeficiente de correlación]
- coefficient of determination [coeficiente de determinación]
- causation [causalidad]
- necessary condition [condición necesaria]
- sufficient condition [condición suficiente]
- common response [respuesta común]
- confounding variable [variable de confusión]
- conjecture [conjetura]
- first differences
- average rate of change
- point-slope form
- standard form [forma estándar/general]
- polynomial [polinomio]
- degree
- leading coefficient
- zero of a function [cero de una función]

Refer to the Math Glossary for definitions of the New Key Terms.

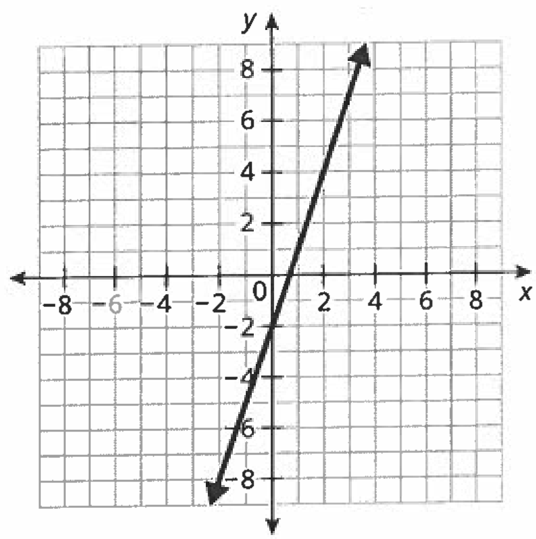
Where are we Now?

The **Least Squares Method** is a method that creates a line that is closest to the points of data, known as a **linear regression function**, for a scatterplot that has two basic requirements: (1) the line must contain the centroid of the data set, and (2) the sum of the squares of the vertical distances from each given data point is smallest with the line.



Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$.

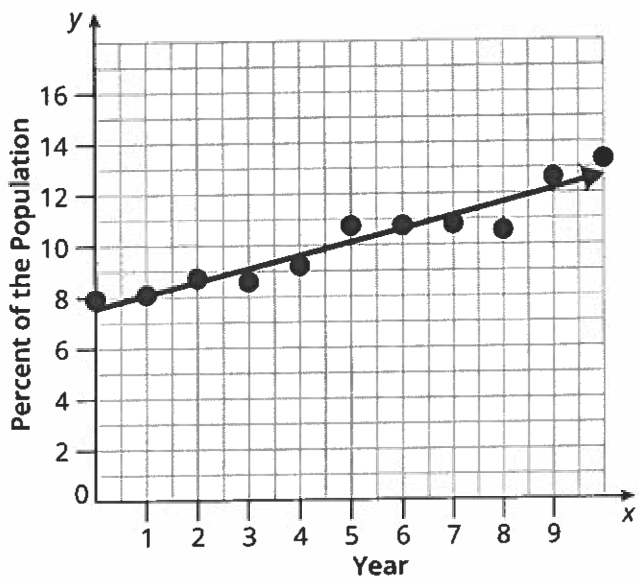
The average rate of change of the function shown is 3.



In **Lesson 1: Least Squares Regressions**, students informally determine a line of best fit before using a formal method to determine a linear regression function. They then use linear regression functions to make predictions.

Regression Lines

Real-world data points usually won't fit neatly on a line. But you can model the data points using a line, which represents a linear function. There are an infinite number of lines that can pass through the data points. But there is just one line that models the data with the minimum distances between the data points and the line. The **linear regression function** has the smallest possible vertical distance from each given data point to the line. By measuring these distances and squaring them, you will see that the squared distances add up to the least value with the linear regression function.



In **Lesson 2: Correlation**, students explore the difference between correlation and causation.

Correlation versus Causation

Consider an experiment conducted by a group of college students that found that more class absences correlated to rainy days. The group concluded that rain causes students to be sick. However, this correlation does not imply **causation**. Rain is neither a **necessary condition** (because students can get sick on days it does not rain) nor a **sufficient condition** (because not every student who is absent is necessarily sick) for students being sick.

In **Lesson 3: Making Connections between Arithmetic Sequences and Linear Functions**, students explore the relationship between the constant difference, slope, and average rate of change.

Slope and Average Rate of Change

The slope, a, of a linear function is equal to the constant difference of an arithmetic sequence. Another name for the slope of a linear function is **average rate of change**. The expression for the average rate of change is $\frac{f(t) - f(s)}{t - s}$. This represents the change in the output as the input changes from s to t.

MYTH

There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: Well, that's one way to do it. Is there another way? What are the pros and cons? That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

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Forms of Linear Equations

In **Lesson 4: Point-Slope Form of a Line**, students use the slope formula to derive the point-slope form of a linear equation.

Students are already familiar with slope-intercept form from previous courses. Other forms they will encounter are **point-slope form** and **standard form**. Each of these has its own special use depending on the given information.

- You can write an equation in point-slope form when given a table.

x	y
2	6
4	5
6	2

- First, use any two points from the table to calculate the slope. For example, use (2, 6) and (4, 5).

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 6}{4 - 2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table. Let's use (2, 6).

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$
$$-\frac{1}{2}(x - 2) = y - 6$$

The equation in point-slope form is $y - 6 = -\frac{1}{2}(x - 2)$.

In **Lesson 5: Using Linear Equations** students use three different forms of a linear equation to graph linear relationships.

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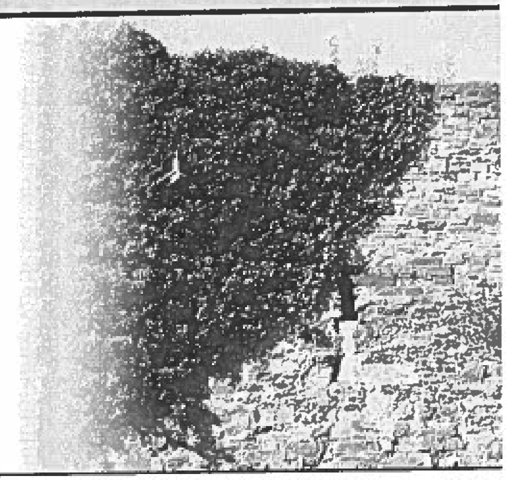
Algebra I

TOPIC 2 Transforming and Comparing Linear Functions



In this topic, students are introduced to function transformations, using vertical and horizontal dilations, and horizontal and vertical translations. Students use translations to prove that the slopes of parallel lines are the same; they use rotations to prove that the slopes of perpendicular lines are negative reciprocals. Understanding the rules of transformations for linear functions lays the groundwork for students to transform any function type.

Where have we been?

Over the last few years, students have had extensive experience with linear relationships. They have represented relationships using tables, graphs, and equations. They understand slope as a unit rate of change, and as the steepness and direction of a graph.

Where are we going?

From this topic, students should understand how to transform linear functions. This prepares students to transform other function types in this course and future courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Linear functions can be an important topic to know about for representing real-world situations and for admissions tests.

HERE IS A SAMPLE QUESTION

x	0	1	2	3
$f(x)$	−2	3	8	13

What equation could represent $f(x)$?

Students may recognize that since the x-values increase by 1s, they can use *first differences* to determine the slope.

$3 - (-2) = 5$ $\quad$ $8 - 3 = 5$ $\quad$ $13 - 8 = 5$

The constant difference is 5, so the slope is 5. The y-intercept is $(0, -2)$, so the equation that represents the function is $y = 5x - 2$.

NEW KEY TERM

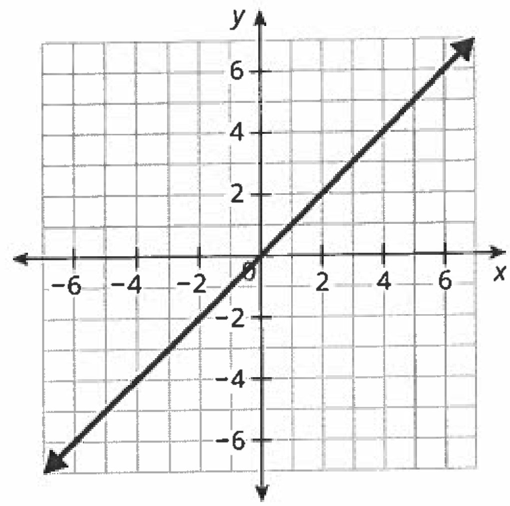
- parent function

Refer to the Math Glossary for definitions of the New Key Terms.

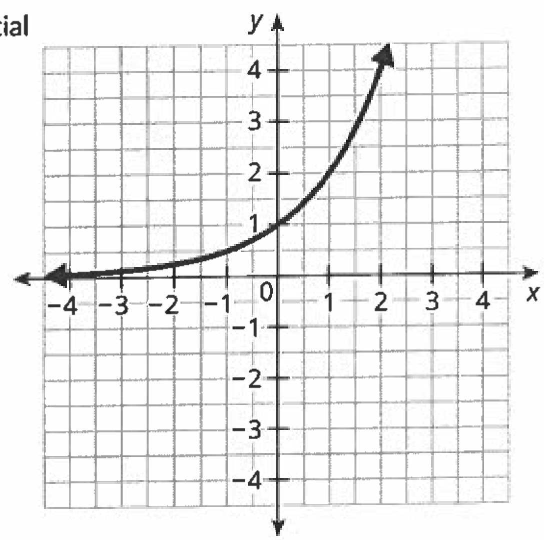
Where are we now?

A **parent function** is the simplest function of its type.

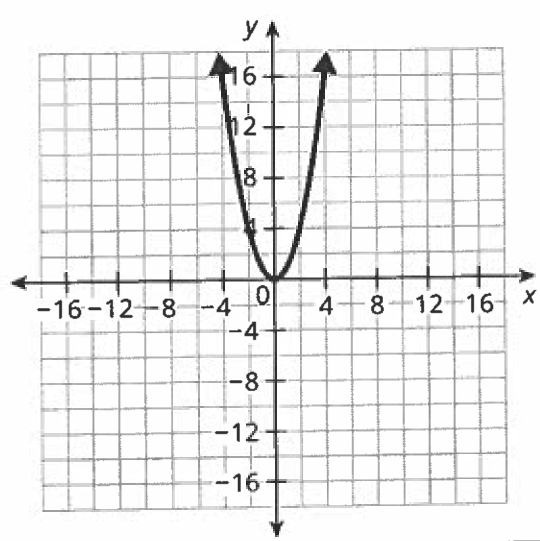
The parent linear function is $f(x) = x$.



The parent exponential function is $g(x) = 2^x$.



The parent quadratic function is $h(x) = x^2$.



In **Lesson 1: Transforming Linear Functions**, and **Lesson 2: Vertical and Horizontal Transformations of Linear Functions**, students are introduced to transformation form.

Transformation Notation

The graph of the function $f(x) = x$ is a straight diagonal line that passes through the origin. When a constant d is added, $g(x) = f(x) + d$, the graph shifts up or down vertically. When the function is multiplied by a constant a, $g(x) = a \cdot f(x)$, the graph is stretched or compressed vertically. When a constant c is added to the input of the function, $g(x) = f(x - c)$, the function shifts left or right horizontally. When the input of the function is multiplied by a constant b, $g(x) = f(bx)$, the graph is stretched or compressed horizontally.

Vertical dilation by a factor of a; when $a < 0$, a reflection across the x-axis.

Horizontal dilation by a factor of $\frac{1}{|b|}$; when $b < 0$, a reflection across the y-axis.

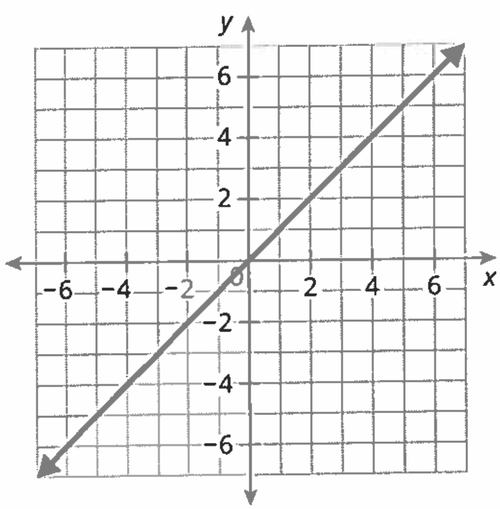
$$a \cdot f(b(x - c)) + d$$

Horizontal translation of c units; to the right when $c > 0$ or to the left when $c < 0$.

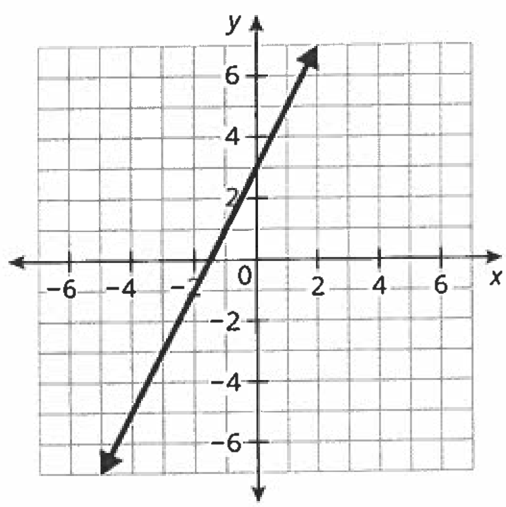
Vertical translation of d units; up when $d > 0$ or down when $d < 0$.

For example, the graph of $y = 2x + 3$ represents both a vertical translation of 3 units and vertical dilation by a factor of 2.

$y = x$



$y = 2x + 3$



MYTH

Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of some science fiction movies that ask the audience: Imagine what you could accomplish if you could use 100% of your brain!

Well, this isn't Hollywood. The good news is that you do use 100% of your brain. As you look around the room, your visual cortex is busy assembling images, your motor cortex is busy moving your neck, and all of the associative areas recognize the objects that you see. Meanwhile, the corpus callosum, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts... like mathematics!

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