MODULE
1

# Searching for Patterns

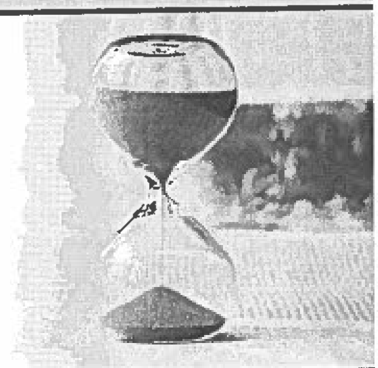
# Family Guide

MODULE 1 Searching for Patterns

Algebra I

## TOPIC 1 Quantities and Relationships

In this topic, students explore a variety of different functions. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study in this course, they will formalize their understanding of the defining characteristics of each type of function.



### Where have we been?

In previous grades, students defined a function and used linear functions to model the relationship between two quantities. They have written linear functions in slope-intercept form and should be able to identify the slope and $y$-intercept in the equation. Students have also characterized graphs as functions using the terms *increasing, decreasing, constant, linear,* and *nonlinear.*

### Where are we going?

The study of functions is a main focus of Algebra I and future math courses. This topic builds the foundation for future, more in-depth study by familiarizing students with the concept of a function. Students will continue to use formal function notation throughout this course and in higher-level math courses.

### *TALKING POINTS*

**DISCUSS WITH YOUR STUDENT**

Functions are an important topic to know for making predictions in the sciences, creating computer programs, and college admissions tests.

### HERE IS A SAMPLE QUESTION

**For the function $f(x) = 2x^2 - 3x$, what is the value of $f(-5)$?**

To solve this, students need to know that the input $-5$ is substituted for $x$ in the equation:

$$\begin{aligned} f(-5) &= 2(-5)^2 - 3(-5) \\ &= 2(25) + 15 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

The point $(-5, 65)$ is on the graph of the function.

## NEW KEY TERMS

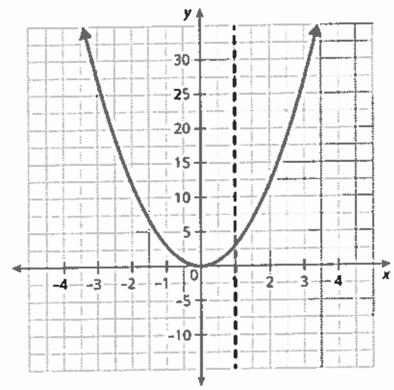
- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- quadratic functions [funciones cuadráticas]
- x-intercept [intersección con el eje x]
- y-intercept [intersección con el eje y]

Refer to the Math Glossary for definitions of the New Key Terms.

## Where are we now?

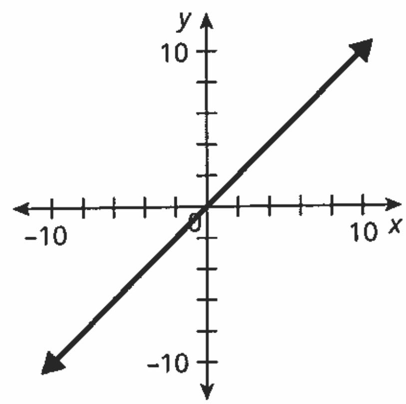
The **Vertical Line Test** is a way to determine if a relation on a graph is a function.

The equation $y = 3x^2$ is a function. The graph passes the vertical line test because there are no vertical lines you can draw that would cross the graph at more than one point.



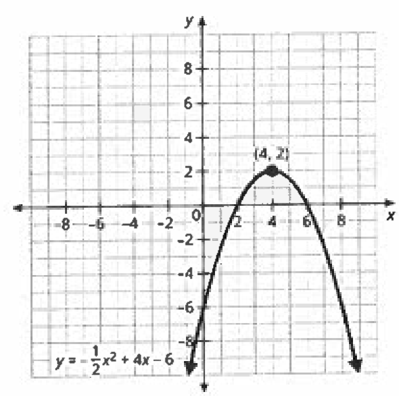

A **continuous graph** is a graph of points connected by a line or smooth curve. Continuous graphs have no breaks.

The graph shown is a continuous graph.




A function has an **absolute maximum** when there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph. It is the highest point that the curve reaches on the graph.

The absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$ is $y = 2$.

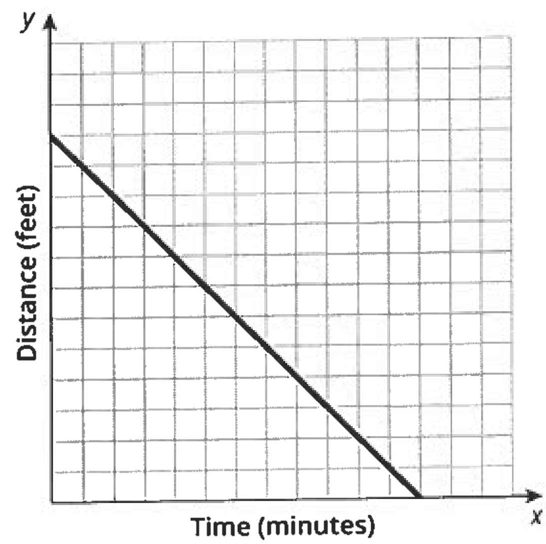


In **Lesson 1: Understanding Quantities and Their Relationships**, students read descriptions of relationships between two quantities and identify which is independent and which is dependent.

## Dependent and Independent Quantities

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**, which is typically represented by the variable y. The quantity that changes the other quantity is called the **independent quantity**, which is typically represented by the variable *x*.

For example, consider the graph that models the situation where Pedro is walking home from school at a constant rate. The time, in minutes, that Pedro walks is the independent quantity. The distance away from home is the dependent quantity.

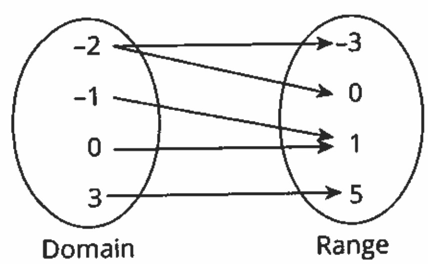



In **Lesson 3, Recognizing Functions and Function Families**, students investigate relations, functions, and function notation.

## Functions and Relations

A **relation** is a mapping between a set of input values called the **domain** and a set of output values called the **range**. A **function** is a relation between a given set of elements, where each element in the domain is grouped with exactly one element in the range. If each value in the domain has one and only one range value, like Figure 2, then the relation is a function. If any value in the domain has more than one range value, like Figure 1, then the relation is not a function.

Figure 1




The value –2 in the domain has more than one range value. The mapping does not represent a function.

Figure 2

| Domain | Range |
|---|---|
| 2 | 1 |
| 6 | 3 |
| 10 | 5 |
| 14 | 7 |

Each element in the domain has exactly one element in the range. The table represents a function.

**MYTH**

***"I don't have the math gene."***

Let's be clear about something. There isn't *a* gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to it.

Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your student's mathematical growth, attend to the learning environment. You can support this by discussing math in the real world, offering encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and providing space for plenty of practice.

**#mathmythbusted**

## Function Notation

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions with algebra. This form allows you to more easily identify the independent and dependent quantities. The function $f(x)$ is read as "$f$ of $x$" and shows that $x$ is the independent variable.

$$f(x) = 8x + 15$$

name of function

independent variable

The linear equation $y = 8x + 15$ can be written to represent a relationship between the variables $x$ and $y$. You can write this linear equation as a function with the name $f$ to represent it as a mathematical object that has a specific set of inputs (the domain of the function) and a specific set of outputs (the range of the function).

## Function Families

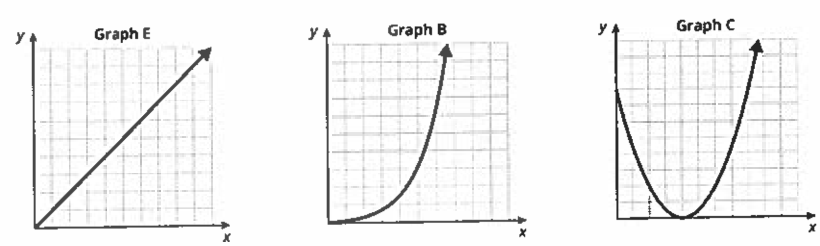
In **Lesson 4: Recognizing Functions by Characteristics**, students associate function families with specific sets of characteristics.

A **function family** is a group of functions that share certain properties. Function families have key properties that are common among all functions in the family. Knowing these key properties is useful when sketching a graph of the function.

The graph of a **linear function** is represented by a straight line which can be vertical, horizontal and diagonal.

The graph of an **exponential function** is represented by a smooth curve.

The graph of a **quadratic function** is represented by a parabola.



# Family Guide

MODULE 1 Searching for Patterns

Algebra I

## TOPIC 2 Sequences

In this topic, students explore sequences represented as lists of numbers, in tables of values, by equations, and as graphs on the coordinate plane. Students move from an intuitive understanding of patterns to a more formal approach of representing sequences as functions.

### Where have we been?

Students have been analyzing and extending numeric patterns since elementary school. They have discovered and explained features of patterns. They have formed ordered pairs with terms of two sequences and compared the terms. In previous grades, students have connected term numbers and term values as the inputs and outputs of a function.

### Where are we going?

As students deepen their understanding of functions throughout this course and beyond, recognizing that all sequences are functions is an important building block. A rich understanding of arithmetic sequences is the foundation for linear functions.

### TALKING POINTS

**DISCUSS WITH YOUR STUDENT**
Sequences are an important topic to know about for college admissions tests.

### HERE IS A SAMPLE QUESTION

What is the second term in this geometric sequence?

$\frac{1}{3}, \_\_, \frac{1}{48}, \frac{1}{192}, \ldots$

To solve this, students need to know that each term in a geometric sequence is calculated by using the same multiplier, or constant ratio. The multiplier can be determined by dividing a term by the term before it.

In this case, $192 \div 48 = 4$. Therefore, $\frac{1}{192} \div \frac{1}{48} = \frac{1}{4}$. This means the multiplier is $\frac{1}{4}$. The second term can be calculated by multiplying the first term by $\frac{1}{4}$.

Because $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$, the second term is $\frac{1}{12}$.

## NEW KEY TERMS

- sequence [secuencia/sucesión]
- term of a sequence [término de una secuencia]
- infinite sequence [secuencia infinita]
- finite sequence [secuencia finita]
- arithmetic sequence [secuencia aritmética]
- common difference [diferencia común]
- geometric sequence [secuencia geométrica]
- common ratio [razón común]
- recursive formula [fórmula recursiva]
- explicit formula [fórmula explícita]
- mathematical modeling [modelado matemático]

Refer to the Math Glossary for definitions of the New Key Terms.

## Where are we now?

A **sequence** is a pattern that has an ordered arrangement of numbers, geometric figures, letters, or other objects.

A **term in a sequence** is an individual number, figure, or letter in the sequence.

A **recursive formula** expresses each new term of a sequence based on the term that comes before it in the sequence.
The recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$.
The recursive formula for a geometric sequence is $g_n = g_{n-1} \cdot r$.

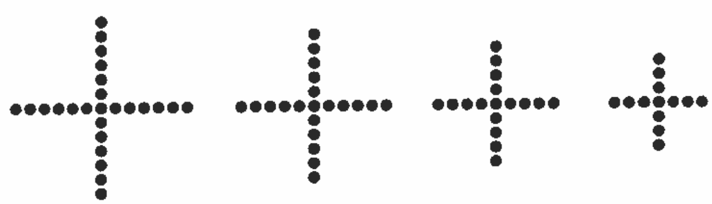
The formula $a_n = a_{n-1} + 2$ is an example of a recursive formula. Each term that comes next is calculated by adding 2 to the previous term. If $a_1 = 1$, then $a_2 = 1 + 2 = 3$.

In **Lesson 1, Recognizing Patterns and Sequences**, students write numeric sequences to represent geometric patterns or contexts.

## Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Consider the pattern shown:



In this pattern, each figure has four fewer dots than the image before. To extend this pattern, draw the following images:

The sequence representing these figures is: 25, 21, 17, 13, 9, 5, 1.

In **Lesson 2, Arithmetic and Geometric Sequences**, students will categorize sequences based on characteristics in the expression.

## Arithmetic Sequence

An **arithmetic sequence** is a sequence of numbers where the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers where a constant is added to each term to produce the next term. This constant is called the **common difference.** The common difference is typically represented by the variable, *d*.

Consider the sequence shown.

$$11, 9, 7, 5, \ldots$$

The pattern is to add the same negative number, −2, to each term to define the next term.

$$\text{Sequence: } \underline{\ 11\ } \xrightarrow{\text{add } -2} \underline{\ 9\ } \xrightarrow{\text{add } -2} \underline{\ 7\ } \xrightarrow{\text{add } -2} \underline{\ 5\ }, \ldots$$

The sequence is arithmetic and the common difference, *d*, is −2.

## Geometric Sequences

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is represented by the variable *r*. For example, in the sequence $27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}$, the pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. For that reason, this sequence is geometric and the common ratio, *r*, is $\frac{1}{3}$.

Geometric Sequence

$$\underbrace{g_n}_{\text{nth term}} = \underbrace{g_1}_{\text{1st term}} \cdot \underbrace{r}_{\text{common ratio}}^{\overbrace{n-1}^{\text{previous term number}}}$$

**MYTH**

***Asking questions means you don't understand.***

It is universally true that, for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. But there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more about a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, everyone should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

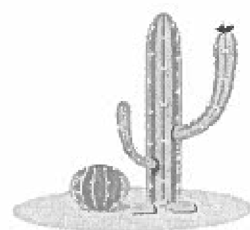
#mathmythbusted

In **Lesson 3, Determining Recursive and Explicit Expressions from Contexts**, students will use and define explicit formulas.

## Explicit Formulas

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the term for any number that you put in the place of $n$ (also known as the "nth term") of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. You can use the distributive property to rewrite the formula for an arithmetic sequence. The explicit formula to determine the $n$th term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

| | | |
|---|---|---|
| $a_n = 3 + \frac{1}{4}(n - 1)$<br>$a_n = 3 + \frac{1}{4}n - \frac{1}{4}$<br>$a_n = \frac{1}{4}n + 2.75$<br>$a_{12} = \frac{1}{4}(12) + 2.75$<br>$a_{12} = 3 + 2.75$<br>$a_{12} = 5.75$ or $5\frac{3}{4}$ |  | In 12 months, the cactus will be $5\frac{3}{4}$ inches tall. |